S-2637/300

S-2637

M. Sc. I (Sem. I) Examination

March / April – 2011

Mathematics

(Fourier Analysis)

Time : **3** Hours] [Total Marks : **70**

Instructions :

(1)

નીચે દર્શાવેલ ☛ નિશાનીવાળી વિગતો ઉત્તરવહી પર અવશ્ય લખવી.
Fillup strictly the details of ☛ signs on your answer book.

Name of the Examination :
☛ **M. SC. 1 (SEM. 1)**

Name of the Subject :
☛ **MATHEMATICS**

☛ Subject Code No. : **2 6 3 7** ☛ Section No. (1, 2,.....) : **NIL**

Seat No. : | | | | | | |

Student's Signature

(2) Attempt all questions.
(3) Follow the usual notations and conventions.
(4) Figures on the **right** indicate full marks.

1 Attempt any **two** from the following : **14**

(a) Find the fourier series of the function $f(x) = x^3$ in the interval $\frac{-\pi}{2} < x < \frac{3\pi}{2}$.

(b) Find the fourier series of the function $f(y) = e^{ay}$ in the interval $-\pi < y < \pi$.

(c) Find the fourier series of the function defined as. $f(x)\begin{cases} 2+x \text{ } for \text{ } -2 \le x \le 0 \\ 2-x \text{ } for \text{ } 0 \le x \le 2 \end{cases}$ where , $f(x+4) = f(x)$..

2 Attempt any **two** from the following : **14**

(a) Find the Half-range cosine series for the function $f(x) = \sin x$ in the interval $0 < x < \pi$.

(b) Find the complex form of the fourier series of $f(x) = c\text{os}\,ax \text{ } in - \pi < x < \pi$.

(c) obtain the fourier series for the function .

$$f(x)=\begin{cases}1+\dfrac{2x}{\pi} & ,-\pi\le x<0\\ 1-\dfrac{2x}{\pi} & ,\ 0\le x\le \pi\end{cases}$$

and hence deduce that $\dfrac{1}{1^2}+\dfrac{1}{3^2}+\dfrac{1}{5^2}+---=\dfrac{\pi^2}{8}$.

3 Attempt any **two** from the following: **14**

(a) Find the fourier integral representation of the function

$$f(x)=\begin{cases}0 & , \ x<0\\ 1 & , \ 0\le x\le 1\\ 0 & , \ x>1\end{cases}$$ and hence show that

$$\int_0^{\infty}\frac{\sin\left(x/2\right)}{x}\,dx=\frac{\pi}{2}\ .$$

(b) Find the sine and cosine transform of the function

$$f(x)=\begin{cases}e^{2x}-e^{-2x} & , \ for\ \ 1\le x<2\\ 0 & , \ for\ \ otherwise\end{cases}$$

(c) Derive the fourier integral formula from fourier series of a function $f(x)$ defined in the interval $-a<x<a$.

4 Attempt any **two** from the following : **14**

(a) Explain the seperation of variable technique to solve the one-dimensional wave equation with respect to the following conditions :-

u(o,t) = u(l,t) = 0 where , $0\le x\le l.$

and $u(x,o)=f(x)$; (u_t); $(x,0)=g(x)$ where, $f(x)$ and $g(x)$ are any arbitrary functions.

(b) Solve the initial Boundary value problem corresponding to one dimensional wave equation

$u_{tt}=c^2u_{xx};0<x<\pi,t>0$ under the conditions :

u(o,t) = $u(\pi,t)=0$, $u(x,0)=0$ and $(u_t)_{(x,0)}=\sin x.$

(c) Solve the initial Boundary value problem $u_{tt} = c^2 u_{xx}$; $0 < x < 2$, $t > 0$ under the condtitions :

$u(0,t) = u(2,t) = 0$ and. $u(x,0) = \begin{cases} x & , 0 \le x < 1 \\ 2-x & , 1 \le x \le 2 \end{cases}$;

$(u_t)_{(x,0)} = 0$.

5 Attempt any **two** from the following : **14**

(a) Solve the heat equation $u_t = u_{xx}$, $x > 0 , t > 0$ subject to the conditions :-

(i) $u_x = (0,t) = 0$

(ii) $u(x,0) = \begin{cases} x & , 0 \le x < 1 \\ 0 & , x > 1 \end{cases}$.

(iii) $u,(x,t)$ is bounded where $x > o, t > 0$ by fourier transform method.

(b) find the fourier series solution of the heat equation $u_t = C^2 u_{xx}$, $0 < x < \pi$, $t > 0$, under the boundary conditions $u(0,t) = u(\pi,t) = 0$ and the initial conditions

$$u(x,0) = \begin{cases} x ; 0 \le x \le \pi/2 \\ \pi - x ; \frac{\pi}{2} < x \le \pi \end{cases}$$

(c) Explain the separation of variable technique to solve the one-dimensional heat equation with respect to the following conditions:-.
$u(0,t) = u(l,t) = 0$ where $0 \le x \le l$ and $u(x,0) = f(x)$.